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ARTICLE

# Numerical Simulation and Experimental Verification of Springback in Magnesium Alloy V-shaped Roll-Bending Using Optimized Hill'48 Yield Criterion

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**Abstract:** The bending springback of magnesium alloys is difficult to predict accurately by numerical simulations because of their anisotropic characteristics. The springback of magnesium alloy V-shaped roll-bending was analyzed using the error optimization function in Matlab to optimize the anisotropic potential values required for the Hill'48 yield criterion in ABAQUS. The optimized Hill'48 yield criterion model was used to numerically simulate the springback of magnesium alloy V-shaped roll-bending. The simulation results were compared with the experimental results. Results show that the error between the springback change ratio obtained using the optimized Hill'48 yield criterion and experimentally formed parts is within 2%. Overall, the optimized Hill'48 yield criterion improves the prediction accuracy of springback in magnesium alloy V-shaped roll-bending.

**Key words:** magnesium alloy; roll-bending; springback; Matlab; anisotropic potential values

## 1 Introduction

Magnesium alloys, characterized by low density, high specific strength, excellent vibration resistance, and good heat dissipation, are widely used in automobile bodies<sup>[1-2]</sup>. As an energy-efficient metal forming technology with broad application prospects, roll-forming is one of the most employed methods for mass production<sup>[3]</sup>. The roll-forming process is expected to become an effective approach for the mass production of magnesium alloy structural parts in the future because of the characteristics of magnesium alloys and mature processing techniques. However, springback, an unavoidable defect in roll-forming processes, significantly

affects the forming profile of the final sheet<sup>[4]</sup>. Therefore, the accurate prediction of the roll-forming springback of the magnesium alloy material and the development of effective control strategies for springback have become critical challenges in the manufacturing of complex magnesium alloy structural parts.

In recent years, the springback in profile bending has been investigated. Gattmeh et al<sup>[5]</sup> used a three-dimensional explicit/dynamic finite element model to analyze the bending process of V-shaped sheets. The effects of the punching radius and sheet thickness on springback and residual strain were predicted. Results showed that the springback decreased as the punching radius decreased and sheet thickness increased.

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Furthermore, the residual strain on the tensile side was greater than that on the compressive side. Sen et al<sup>[6]</sup> explored the forming properties of CP800 sheets under V-shaped bending conditions by combining experiments with finite element analysis. The springback magnitudes under different bending angles were obtained. Nie et al<sup>[7]</sup> used a combination of experiments and finite element simulations to study the springback of titanium alloy during the V-shaped hot bending unloading. Ning et al<sup>[8]</sup> examined the diversity of microstructure, springback, and texture of AZ31B magnesium alloy sheet during continuous room-temperature bending at three loading rates. Results showed that the springback was minimized at a bending rate of 100 mm/min. Furthermore, the experimental process was simulated accurately using finite element software.

The numerical simulation techniques are widely used in practical engineering applications due to their capability to accurately predict the final geometrical features, mechanical properties, and defects generated during molding<sup>[9]</sup>. These predictions are crucial for guiding subsequent experimental investigations. The yield function model greatly influences the accuracy of finite element analysis, especially for sheets with unique behavior. Many studies have been conducted to develop yield function model that better capture the material behavior. Moreover, new yield function models have been proposed<sup>[10]</sup>. The von Mises criterion describes the yield behavior for isotropic materials. However, it requires some additional parameters for anisotropic materials. To address this, the Hill yield criterion was proposed in 1948<sup>[11]</sup> and is now widely implemented in finite element analysis software such as ABAQUS for anisotropic material modeling<sup>[12]</sup>. Wang et al<sup>[13]</sup> obtained the mechanical properties and anisotropic parameters of Al-Mg-Li alloys through uniaxial and biaxial tensile tests and ultimate strain tests. They predicted the forming limit curves of the sheets by combining the modified M-K model with the Yld2000-2d yield criterion, which was verified experimentally. Yan et al<sup>[14]</sup> proposed an inverse parametric method to determine the Hill'48 yield criterion parameters based on plane strain tensile experiments combined with finite element analysis. Results showed that the Hill'48 yield criterion predictions using the obtained parameters were superior to the von Mises criterion predictions. Trieu et al<sup>[15]</sup> compared Hill'48S, Hill'48R, and von Mises yield criteria for predicting anisotropic behavior and fracture in SECC steels. The results showed that the fracture predicted via Hill'48R yield criterion was closer to the experimental results. Furthermore, the study also highlighted the importance of each potential anisotropy value ( $r$ -parameters) and advocated a direction for further research in this area.

Against this backdrop, the present study proposes an innovative method to optimize the Hill'48 yield criterion parameters. This method aims to improve the prediction accuracy of numerical simulation for the roll-bending process while minimizing the number of basic experiments required to obtain more accurate Hill'48 yield criterion parameters. The error optimization function was introduced, and the optimal anisotropic potential values were obtained via iterative optimi-

zation using Matlab. Subsequently, a finite element model was established based on the optimized Hill'48 yield criterion for the V-shaped roll-bending of magnesium alloy sheets. The numerical simulation results were experimentally verified.

## 2 Experiment

AZ31B magnesium alloy was used in this study. Table 1 lists the chemical composition of the alloy. It exhibits superior mechanical properties compared with steel and is mainly used in aerospace, automotive industry, communications, weapons, and other fields.

Uniaxial tensile experiments of magnesium alloy were conducted using an electronic universal tensile testing machine. According to the test requirements, uniaxial tensile specimens of AZ31B magnesium alloy were prepared along three typical directions: the rolling direction (RD), 45° to RD (45°-RD), and the transverse direction (TD, perpendicular to the rolling direction). Fig. 1 shows the tensile specimens in each testing direction and their dimensions.

The size of the magnesium alloy sheet used in the V-shaped roll-bending experiment was 700 mm×78 mm×2 mm. The forming angle of the three passes was 0°, 15°, and 25°, and the roll gap was 2.0, 2.2, and 2.5 mm. The distance between machine frames was 480 mm.

## 3 Numerical Simulation

In ABAQUS finite element simulation software, the von Mises criterion is widely used<sup>[16]</sup>. However, for hcp magnesium alloy with anisotropy, the embedded von Mises yield criterion in the software cannot accurately describe the bending behavior of magnesium alloy sheets<sup>[17]</sup>. Regarding the anisotropic characteristics of magnesium alloy, various anisotropic yield criteria are proposed, such as CaBa2004 yield criterion, CPB06 yield criterion, and Hill yield criterion. This study focuses on the Hill yield criterion.

### 3.1 Method for solving anisotropic parameters of the Hill'48 yield criterion

The thickness anisotropy coefficient  $r$  expresses the deformation anisotropy characteristics of magnesium alloy sheets. It reflects the difference between the plastic

Table 1 Chemical composition of AZ31B magnesium alloy (wt%)

Al	Si	Ca	Zn	Mn	Fe	Cu	Ni	Mg
3.2	0.08	0.04	1.4	0.7	0.03	0.01	0.001	Bal.

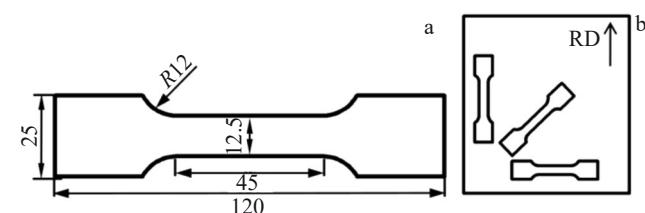


Fig.1 Uniaxial tensile test specimens

deformation along the width and thickness directions during the tensile testing, and it can be calculated using Eq.(1).

$$r = \frac{\varepsilon_w}{\varepsilon_t} \quad (1)$$

where  $\varepsilon_w$  is the strain in the width direction and  $\varepsilon_t$  is the strain in the thickness direction.

For each anisotropic material,  $r$  values along the RD, 45°-RD, and TD were obtained by solving Eq.(1), and the three obtained values were substituted into Eq.(2-5) to obtain the anisotropic parameters for the Hill'48 yield criterion<sup>[18]</sup>.

$$F = \frac{r_0}{(1 + r_0)r_{90}} \quad (2)$$

$$G = \frac{1}{1 + r_0} \quad (3)$$

$$H = \frac{r_0}{1 + r_0} \quad (4)$$

$$N = \frac{(1 + 2r_{45})(r_0 + r_{90})}{2(1 + r_0)r_{90}} \quad (5)$$

where  $F$ ,  $G$ ,  $H$ , and  $N$  are the anisotropy parameters;  $r_0$ ,  $r_{45}$ , and  $r_{90}$  are thick anisotropy coefficients of uniaxial tensile testing along RD, 45°-RD, and TD, respectively.

### 3.2 Solving method of anisotropic potential values for the Hill'48 yield criterion

In ABAQUS, the anisotropic parameters of the Hill'48 yield function under plane stress state can be defined using the following expressions.

$$F = \frac{1}{2} \left( \frac{1}{R_{22}^2} + \frac{1}{R_{33}^2} - \frac{1}{R_{11}^2} \right) \quad (6)$$

$$G = \frac{1}{2} \left( \frac{1}{R_{33}^2} + \frac{1}{R_{11}^2} - \frac{1}{R_{22}^2} \right) \quad (7)$$

$$H = \frac{1}{2} \left( \frac{1}{R_{11}^2} + \frac{1}{R_{22}^2} - \frac{1}{R_{33}^2} \right) \quad (8)$$

$$N = \frac{3}{2R_{12}^2} \quad (9)$$

where  $R_{11}$ ,  $R_{22}$ ,  $R_{33}$ , and  $R_{12}$  are the anisotropic potential values.

### 3.3 Optimization of anisotropic potential values for the Hill'48 yield criterion

Generally, obtaining precise anisotropic potential parameters of the Hill'48 yield criterion requires multiple experiments, which are often time-consuming. Considering the constraints of experimental conditions and time, this study proposes an optimization method for the anisotropic potential parameters of the Hill'48 yield criterion to improve the accuracy of simulation results.

To optimize the anisotropic potential parameters, the theoretical value of the ratio of yield stress in each direction to that in the RD of the material was introduced, which can be obtained using the following equation:

$$R^m(\theta) = \frac{1}{\sqrt{(F+G)\sin^4\theta + (G+H)\cos^4\theta + 2(N-H)\sin^2\theta\cos^2\theta}} \quad (10)$$

where  $\theta$  represents three directions, i.e., 0°, 45°, and 90°.

The results calculated by Eq. (10) tend to exhibit discrepancies with experimental value. To minimize the error, the error optimization function expression was established, as

shown in Eq. (11). Using Matlab to write the error optimization function expression, four anisotropic parameters were iteratively optimized within the range of 0.25–2.00. This process continued until the error between  $R^m$  and  $R^t$  reached the minimum, where  $R^m$  is the theoretical value of ratio of yield stress and  $R^t$  is the experimental value of ratio of yield stress minimum, thereby obtaining the optimal anisotropic parameters. The error optimization function is expressed as:

$$y(F,G,H,N) = (R_0^m - R_0^t)^2 + (R_{45}^m - R_{45}^t)^2 + (R_{90}^m - R_{90}^t)^2 \quad (11)$$

The optimal anisotropic parameters derived from the continuous iteration were substituted into Eq. (6–9) to calculate the optimized anisotropic potential values of the Hill'48 yield criterion. Table 2 lists the results, providing a basis for accurate simulation analysis.

### 3.4 Establishment of roll-bending simulation model

#### 3.4.1 Geometric model

Fig. 2 shows the assembly diagram of three roll-bending forming models. The rolls were set as discrete rigid bodies because the analysis focused on the deformation of the sheet rather than that of the rolls.

#### 3.4.2 Boundary conditions

In the simulation process of roll-bending, the corresponding boundary conditions should be given to replace the roll speed in the experiment.

The roll angular velocity was required in ABAQUS simulation software. The angular velocity calculation formula was as follows:

$$\omega = 2\pi \frac{L}{RT} \quad (12)$$

where  $L$  is the forming path of metal sheet, referring to two machine frame spacings and sheet length;  $R$ ,  $T$ , and  $\omega$  are the radius of the roll, the theoretical time of the roll-bending analysis step, and the angular speed of the forming roll, respectively. In this study,  $L=2000$  mm and  $T=1$  s.

The calculation of roll angular velocity should follow the right-hand principle, i.e., the upper and lower roll angular velocities were positive and negative, respectively. Table 3 lists the radius and angular velocities of each roll in the model.

Table 2 Data of anisotropic potential values

$R_{11}$	$R_{22}$	$R_{33}$	$R_{12}$	$R_{13}$	$R_{23}$
1.3056	1.4142	1.3056	2.4495	1	1

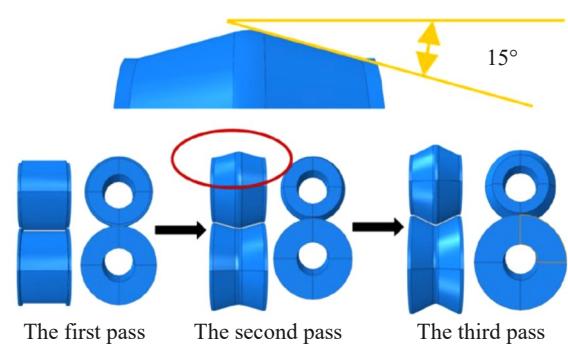


Fig.2 Geometric modeling of V-shaped roll-forming

**Table 3** Roll radius and angular velocity

Roll	Roll radius/mm	Angular velocity/rad·s <sup>-1</sup>
A0	63.0	199.36
A1	64.2	212.92
A2	64.8	223.26
B0	65.0	-193.23
B1	63.8	-182.02
B2	63.2	-175.06

### 3.4.3 Meshing

Due to the influence of longitudinal stress on the solid element during the simulation of roll-bending, it can be easily distorted. Hence, the mesh type of the roll was a discrete rigid body element. The magnesium alloy sheet, S4R, was adopted as a solid shell unit<sup>[19]</sup>. The mesh was refined in the bending angle part of the roll to ensure convergence; the thinning mesh can effectively solve the influence of the hourglass phenomenon. The total number of elements was 12 250. Fig.3 shows the overall mesh division and an enlarged view of the meshing of the upper roll in the second pass.

### 3.5 Analysis methods

Considering the computational efficiency and accurate calculation results, the dynamic explicit algorithm was firstly used to simulate the sheet-forming process. Then, using this result file as the basis, the static implicit algorithm was employed to simulate the sheet springback.

## 4 Results and Discussion

### 4.1 Tensile test results

The tensile test was performed on magnesium alloy sheet to obtain engineering stress and engineering strain, as shown in Fig.4.

### 4.2 Analysis of springback results

#### 4.2.1 Finite element analysis

The edge and middle sections of the forming region of the magnesium alloy sheet are forced to move toward the centerline of the rolls under the action of the upper and lower rolls. Bounded by the neutral layer of the sheet, the inner and outer sides are the compression and tensile regions, respectively. During the roll-bending forming process of the magnesium alloy sheet, the bending part is divided as tensile plastic deformation, compressive plastic deformation, and

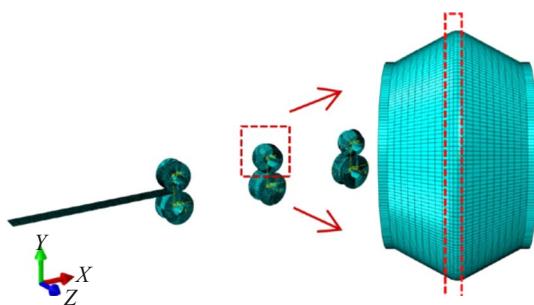


Fig.3 Grid division diagram

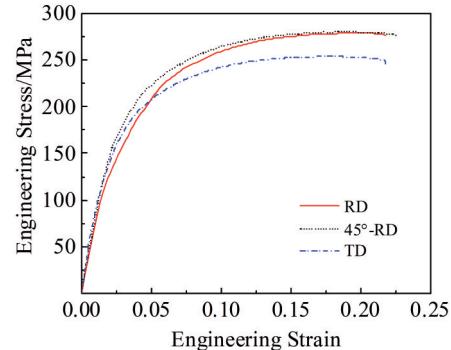


Fig.4 Engineering stress-engineering strain curves

elastic deformation, of which the elastic deformation region is the main cause of springback, as shown in Fig.5.

Fig.6a compares finite element simulation results obtained using the optimized Hill'48 yield criterion model and von Mises criterion model. The sheet cross-section profile with the optimized Hill'48 yield criterion is changed in forming angle compared with that with the von Mises criterion, so the springback situation is different. A larger forming angle indicates a smaller springback<sup>[20]</sup>. The forming angle (Fig.6b) was measured using the Measurement Angle Module in ABAQUS. The final bottom-line-forming angle of the sheet cross-section profile using the optimized Hill'48 yield criterion model is 23.7°, and the springback angle is 1.3°.

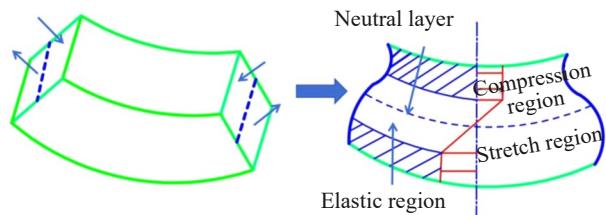


Fig.5 Deformation state of the corners

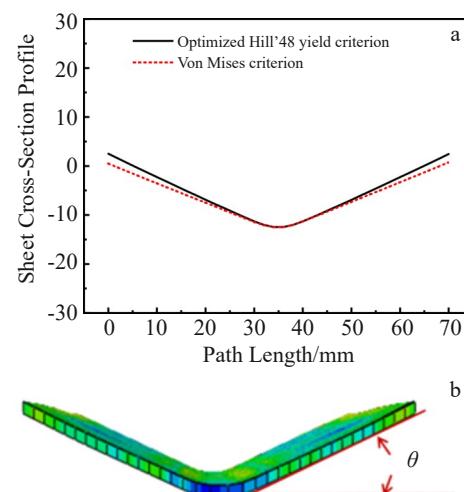


Fig.6 V-shaped sheet profile using different calculation models (a) and springback angle measurement (b)

Although the bottom-line-forming angle is  $21.7^\circ$  under the von Mises criterion, the springback angle is  $3.3^\circ$ . Hence, the sheet springback obtained using the von Mises criterion model is larger than that obtained using the optimized Hill'48 yield criterion model.

#### 4.2.2 Analysis of roll-forming experiment

Fig. 7 shows the formed parts of the V-shaped roll-bending experiment. Using the angle gauge to measure the bottom-line-forming angle of the cross-section profile of formed parts after the roll-forming experiment, an accurate bottom-forming angle result is obtained. The average of the three formed parts was used as the bottom-line-forming angle of the V-shaped roll-formed parts. The final bottom-line-forming angle is  $23.2^\circ$ , and the springback angle is  $1.8^\circ$ .

#### 4.3 Comparative analysis of simulation and experiment results

The results obtained using the optimized Hill'48 yield criterion and von Mises criterion are compared with the experimental result, as shown in Fig.8.

The springback change ratio was calculated using the following formula to compare the springback in three cases:

$$\varphi = \frac{\theta_x - \theta_0}{\theta_0} \quad (13)$$

where  $\theta_x$  is the springback angle obtained from the von Mises criterion simulation analysis, the optimized Hill'48 yield criterion simulation analysis, and the roll-bending experiment;  $\theta_0$  is the theoretical forming angle of the roller in the third pass, equal to  $25^\circ$ . The springback change ratio is obtained using Eq.(13), as shown in Fig.9.

Using the Hill'48 yield criterion optimized by optimizing each anisotropic potential value, the springback change ratio is calculated as 5.2% (Fig.9). However, the springback change



Fig.7 Magnesium alloy V-shaped roll-formed parts

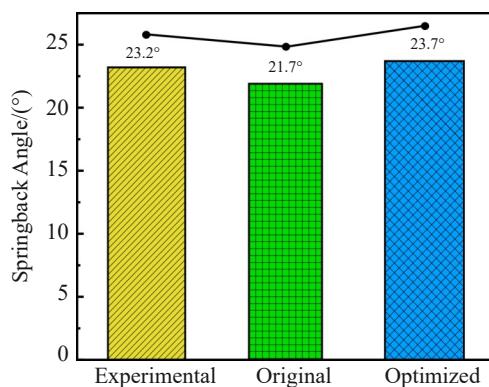


Fig.8 Comparative analysis of the springback angle

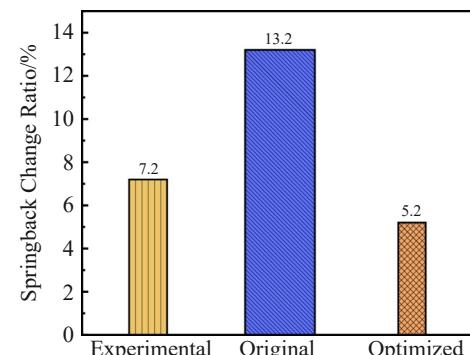


Fig.9 Springback change ratio

ratio obtained by the von Mises criterion is 13.2%, and the springback change ratio of the experimentally formed magnesium alloy parts is 7.2%. The error value for the springback change ratio between the optimized Hill'48 yield criterion and the experimentally formed parts is within 2%. Hence, V-shaped roll-bending numerical simulation analysis of springback using the optimized Hill'48 yield criterion under optimized anisotropic potential values for the magnesium alloy can improve the springback prediction accuracy.

## 5 Conclusions

1) Error optimization function in Matlab software is used to optimize the anisotropic potential values required for simulation by ABAQUS. The V-shaped roll-bending springback model for magnesium alloy is established using the optimized Hill'48 yield criterion.

2) An explicit dynamic algorithm and an implicit static algorithm are used to analyze the V-shaped roll-bending springback for the magnesium alloy. Based on the optimized Hill'48 yield criterion, the bottom-line-forming angle of the formed parts is  $23.7^\circ$ . The theoretical forming angle and the springback change ratio are  $25^\circ$  and 5.2%, respectively. In contrast, the bottom-line-forming angle and the springback change ratio of the numerical simulation using the von Mises criterion are  $21.7^\circ$  and 13.2%, respectively.

3) The springback results in three cases are compared. The error value for the numerical simulation result using the optimized Hill'48 yield criterion and the experiment result is within 2%, confirming the accuracy of the optimized Hill'48 yield criterion model.

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## 采用参数优化后的Hill'48屈服准则对镁合金V型辊弯成形回弹 进行数值模拟研究及实验验证

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**摘要:** 镁合金由于其各向异性特征, 在数值模拟计算中难以准确预测镁合金板材弯曲回弹过程。为了更准确地分析镁合金V型辊弯成形回弹, 利用Matlab应用误差优化函数优化ABAQUS仿真软件中Hill'48屈服准则所需的各向异性势值参数。采用优化后的Hill'48屈服准则模型对镁合金V型辊弯成形回弹进行数值模拟研究, 将其结果与实验结果进行对比。结果表明, 采用优化后的Hill'48屈服准则后回弹变化率与辊弯成形实验成型件的回弹变化率误差在2%以内。采用各向异性势值优化后的Hill'48屈服准则可以提高镁合金V型辊弯成形的回弹预测精度。

**关键词:** 镁合金; 辊弯成形; 回弹; Matlab; 各向异性势值

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